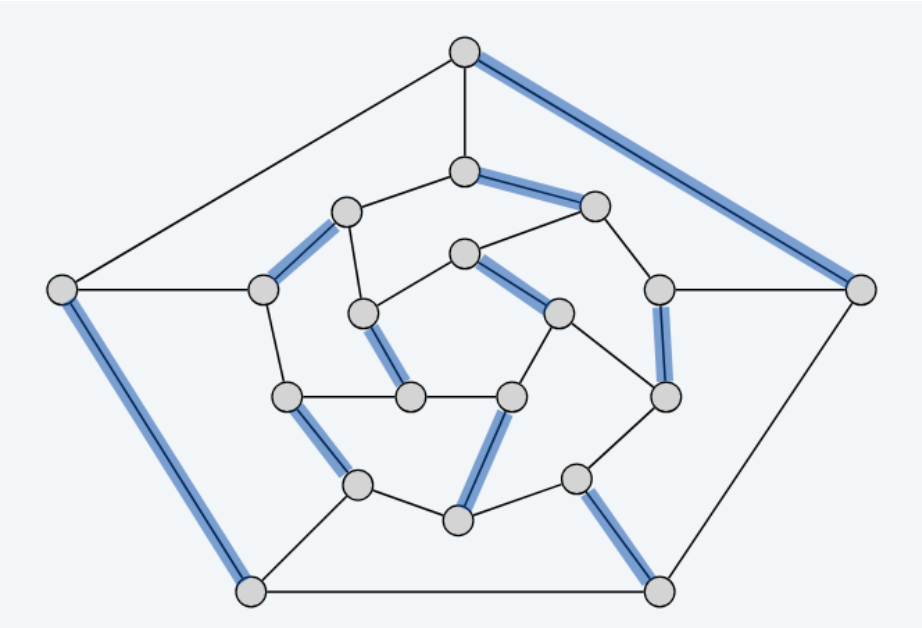
Matchings

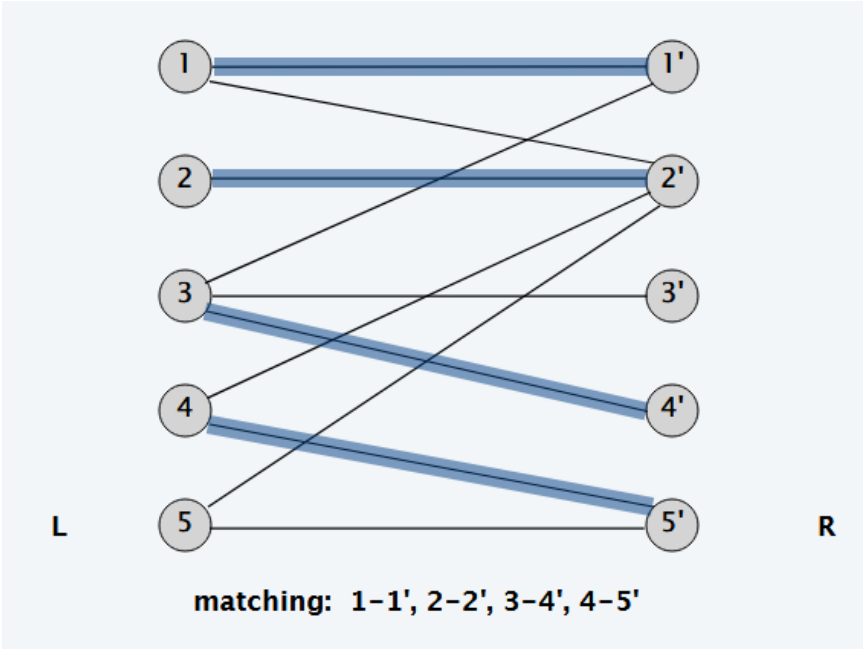
Def. Given an undirected graph G = (V, E), subset of edges $M \subseteq E$ is a matching if each node appears in at most one edge in M.



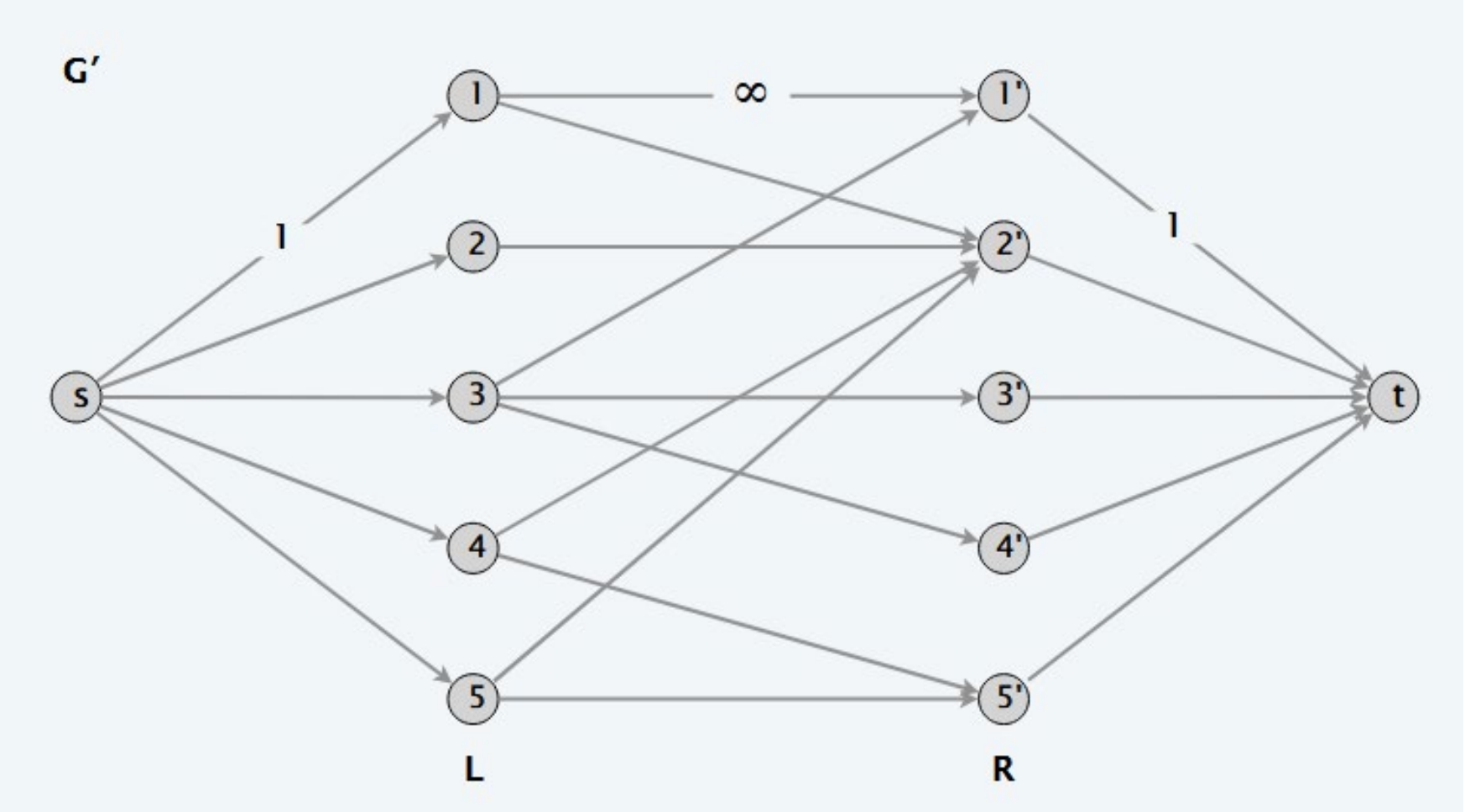
Bipartite Matching

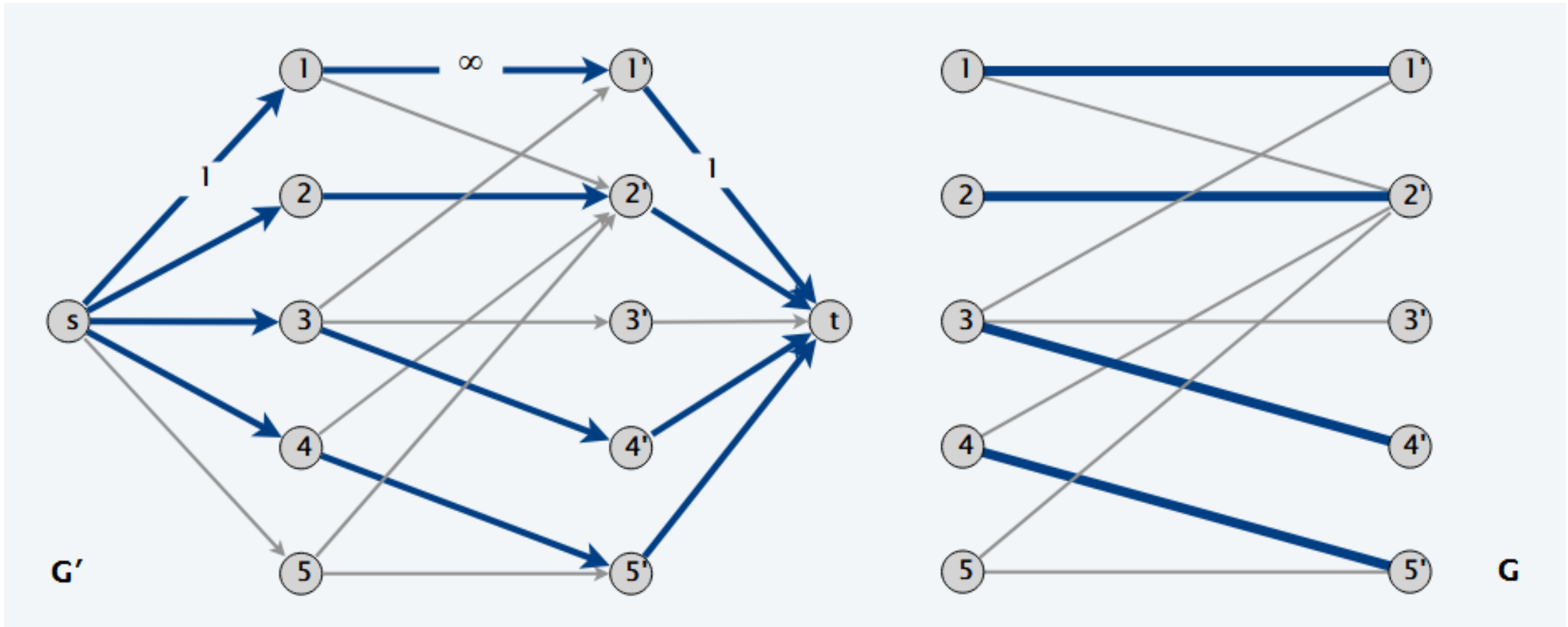
Def. A graph is **bipartite** if the nodes can be partitioned into two subsets L and R such that every edge connects a node in L with a node in R.

Bipartite matching. Given a bipartite graph G = (L ∪ R, E), find a max-cardinality matching.



Reduction to flows

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Perfect Matchings in Bipartite Graphs

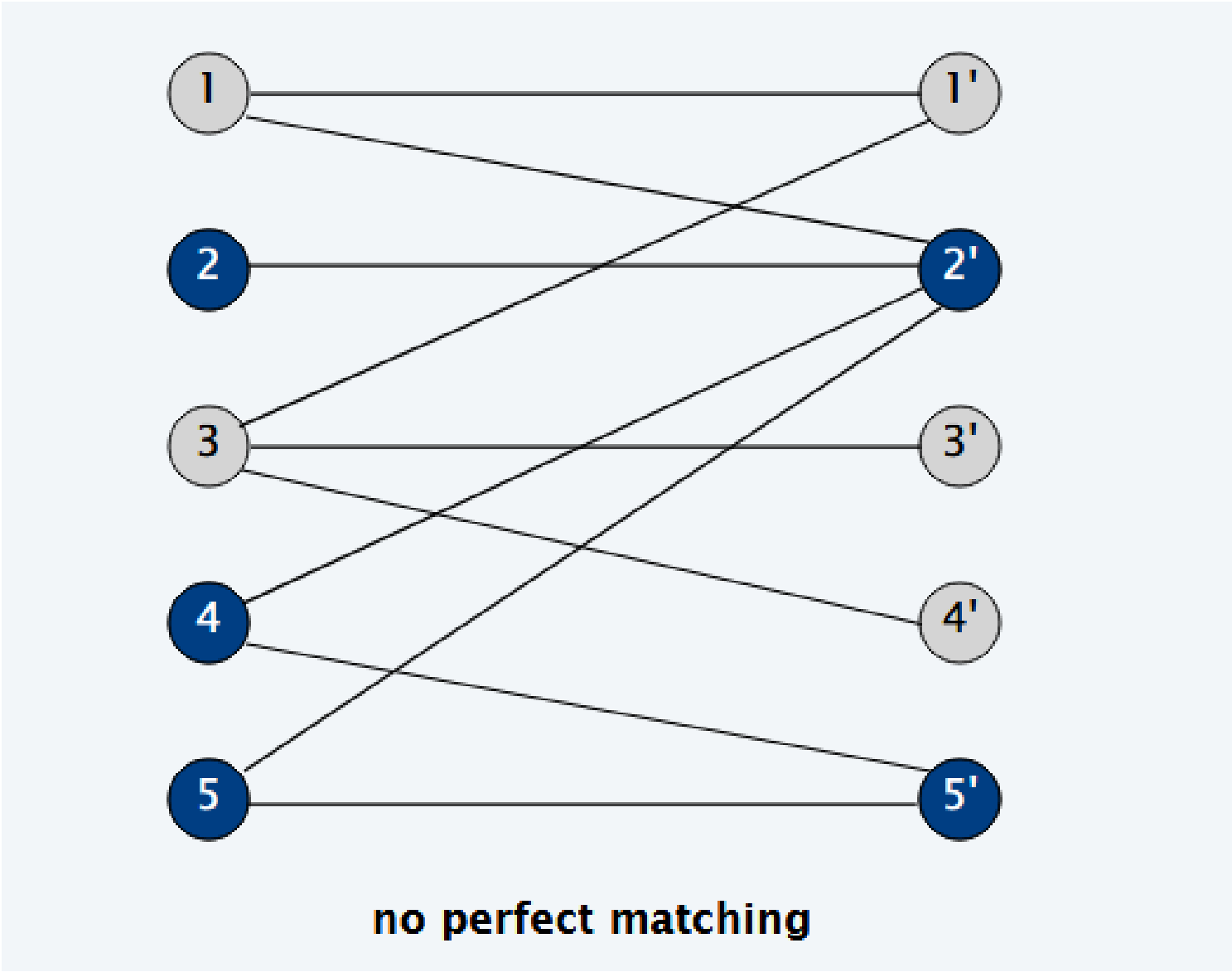
Q. When does a bipartite graph have a perfect matching?

Structure of bipartite graphs with perfect matchings.

- Clearly, we must have |L| = |R|.
- Which other conditions are necessary?
- Which other conditions are sufficient?

Hall Violating set:

The neighborhood of {2,4,5} is {2', 5'}, which is smaller than the set



Hall's Theorem

Theorem: Let $G = (L \cup R, E)$ be a bipartite graph with $|L| = |R|$.

Then G has a perfect matching if and only if $|N(S)| \geq |S|$ for all subsets $S \subseteq L$.

Hackathon problem:

There is a Hackathon:

- attended by n Claremont students and n CMU students.

- each Claremont student is friends with exactly k CMU students

- each CMU student is friends with exactly k Claremont students.

Question:

Is it possible to arrange the hackathon so that each Claremont student pair programs with a friend from CMU?